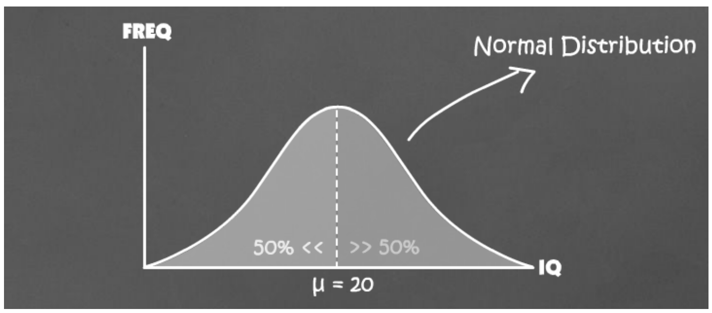
Module 1: Being Normal in a Z-Score World

Topics: Normal Distribution, Standard Deviation, & Z-Scores

Normal Distribution

In this distribution, the IQ scores are plotted on the x-axis, with the lower scores toward the left and the higher scores toward the right. The y-axis represents the frequency of scores. The more frequent a particular score is, the taller its point on the graph will be.



As you can see, this distribution is perfectly bell-shaped. That means this is a *normal distribution*. Normal distributions like this are the most important distributions in statistics.

For a distribution to be normal it needs to be perfectly symmetrical and the scores need to be increasingly frequent toward the middle. In other words, it needs to be bell-shaped and the population needs to be split, with 50% on one side of the mean and 50% on the other side.

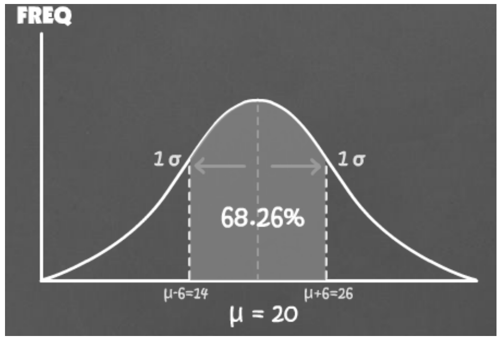
You can calculate how much of the population has a score above and below any point on a normal distribution.

Standard Deviation

Statisticians use standard deviation to measure the degree to which individual scores differ from the mean. The standard deviation of a population is denoted by σ (sigma). The standard deviation of a sample is written as s.

NOTATION:
σ – standard deviation for a population
s – standard deviation for a sample

1 standard deviation from the mean in both directions includes **68.26%** of the population.
(if you took a random individual from the population, you could be 68.26% certain that its score would be within one standard deviation of the mean)



In the example above, one standard deviation unit equals six IQ points. Therefore, since the mean IQ is 20, the range from 14 to 26 IQ points (20 +/- 6) contains 68.26% of the population.

2 standard deviations from the mean in both directions contains **95.44%** of the population.
3 standard deviations from the mean in both directions contains **99.74%** of the population.

In a population with low variability, one standard deviation would include a small range (e.g. +/- 3 points). In a population with high variability, one standard deviation would include a wider range (e.g. +/- 50 points). The higher the variability in the population, the higher the standard deviation.

Remember that, while the value of one standard deviation unit can change depending on the variability of the data, the proportion above and below those standard deviation units will always include the same proportion of the population. For example, **1.96 standard deviations** from the mean in both directions will always include exactly **95%** of the population, regardless of whether the standard deviation equals 10, 20, or even 5,000.

Use the following equations to determine the standard deviation for a population or for a sample.

Standard Deviation
(for populations)

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

σ = standard deviation
N = population size
x_i = value of individual i
μ = population mean

Sample Standard Deviation
(for samples)

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

s = sample standard deviation
n = sample size
x_i = value of individual i
$\bar{x}$ = sample mean

Z-Scores

An individual's z-score tells us how many standard deviations her score is from the mean score. Here are two formulas for calculating a z-score, depending on whether you are calculating a z-score for a population or for a sample.